Example 17 Evaluate the following.
(a) $\frac{-9}{-9}$
(b) $\frac{-9}{\frac{1}{9}}$
(c) $\frac{\frac{1}{9}}{-9}$

Solution
(a) $\frac{-9}{-9}=\frac{9}{9}$
$(-a) \div(-b)=\frac{-a}{-b}=\frac{a}{b}$
= 1
(b) $\frac{-9}{\frac{1}{9}}=-9 \div \frac{1}{9} \quad(-a) \div b=\frac{-a}{b}$

$$
\begin{aligned}
& =-9 \times 9 \quad \frac{1}{9} \text { and } 9 \text { are reciprocals. } \\
& =-81
\end{aligned}
$$

(c) $\frac{\frac{1}{9}}{-9}=\frac{1}{9} \div(-9) \quad a \div(-b)=\frac{a}{-b}$
$=\frac{1}{9} \times\left(-\frac{1}{9}\right) \quad-9$ and $-\frac{1}{9}$ are reciprocals.

$$
=-\frac{1}{81}
$$

Try It! 17 Evaluate the following.
(a) $\frac{3}{-3}$
(b) $\frac{-7}{-\frac{1}{2}}$
(c) $\frac{-\frac{1}{4}}{8}$

Example 18 Find the value of $\left(-2 \frac{2}{5}\right) \times \frac{3}{4}$, giving your answer in its simplest form.

Solution $\quad\left(-2 \frac{2}{5}\right) \times \frac{3}{4}=-\frac{12}{5} \times \frac{3}{4}$

$$
\begin{aligned}
& =-\frac{9}{5} \\
& =-1 \frac{4}{5}
\end{aligned}
$$

Try It! 18 Find the value of $\left(-6 \frac{2}{3}\right) \times \frac{2}{5}$.

## REMARKS

To multiply or divide fractions, the denominators need not be converted to the same number.

### 3.3 Writing Algebraic Expressions to Represent Real-world Situations

We may use algebraic expressions and formulas to express the relationship between two or more quantities in our daily life. Let us see some examples.

Example 12
At birth, baby George weighs 5 ounces lighter than his twin brother Henry.
(a) Suppose Henry's weight is $x$ pounds. What is George's weight in terms of $x$ pounds?
(b) Given that George weighs $y$ pounds, write a formula connecting $x$ and $y$.

Solution

(a) 5 ounces $=\frac{5}{16}$ pounds
$\therefore$ George's weight $=\left(x-\frac{5}{16}\right)$ pounds.
(b) From (a), we have $y=x-\frac{5}{16}$.

Note: When writing algebraic expressions and formulas, check that the variables are representing quantities with similar units. It will then not be necessary to state the units of measurement when writing algebraic formulas.
$y$ pounds $=\left(x-\frac{5}{16}\right)$ pounds is simply written as
$y=x-\frac{5}{16}$.

Try It! 12 Peter is 3 inches taller than Sue.
(a) Suppose the height of Sue is $s$ feet. What is the height of Peter in terms of $s$ feet?
(b) Given that Peter's height is $p$ feet, write a formula connecting $s$ and $p$.

From the above class activity, we observe that $3(x+y)=3 x+3 y$. In general, we have the distributive law of multiplication over addition:

$$
a(x+y)=a x+a y
$$

We say that $a(x+y)$ is expanded to $a x+a y$. The expressions $a(x+y)$ and $a x+a y$ are equivalent expressions.

This law can be generalized and applied as follows:

1. $(x+y) a=a(x+y)$

$$
\begin{aligned}
& =a x+a y \\
& =x a+y a
\end{aligned}
$$

Multiplication can be distributed over addition from the right.
2. $a(x-y)=a[x+(-y)]$

$$
\begin{aligned}
& =a x+a(-y) \\
& =a x-a y
\end{aligned}
$$

Multiplication can be distributed over subtraction.
3. $a(x+y+z)=a x+a y+a z$

Multiplication can be distributed over several terms.

Example 3 Expand each expression by removing the parentheses.
(a) $a(3 b+c)$
(b) $-x(2 y-z)$

Solution (a) $a(3 b+c)=3 a b+a c$

$$
\text { (b) } \begin{aligned}
-x(2 y-z) & =-x[2 y+(-z)] \\
& =(-x)(2 y)+(-x)(-z) \\
& =-2 x y+x z
\end{aligned}
$$

Try It! 3 Expand each expression by removing the parentheses.
(a) $a(2 b-3 c)$
(b) $-x(-5 y+z)$

## RECALI

Note: The distributive law is applicable when removing parentheses in algebraic expressions such as $x-(a-b)$. This expression can be interpreted as $x+(-1)(a-b)$.

$$
\begin{aligned}
x-(a-b) & =x+(-1)(a-b) \\
& =x+(-1)[a+(-b)] \\
& =x+(-1)(a)+(-1)(-b) \\
& =x-a+b
\end{aligned}
$$

Objective: To formulate linear equations to solve problems.

## Questions

A group of boys and girls planted a total of 148 trees. Each boy planted 7 trees, and each girl planted 5 trees. There were 4 more boys than girls in the group.
How many boys were there in the group?

1. Identify the unknown quantity that you are required to find in the problem.
2. Use a letter (e.g. $x$ ) to represent the unknown quantity.
3. Express other quantities in terms of the letter that represents the unknown quantity. In this case, express each of the following in terms of $x$.
(a) the number of girls
(b) the number of trees planted by all the boys
(c) the number of trees planted by all the girls
4. Form an equation required to solve the problem.
5. Solve the equation.
6. Write down the answer statement.

## Alternative Method:

We may use a model to formulate the problem.
Let the number of girls be $y$.

$$
\begin{aligned}
& \text { Girls: } y \\
& \text { Boys: } y \\
& \hline
\end{aligned}
$$

Consider the numbers of trees planted by the girls and the boys, denoted by GT and BT respectively, using the following model.


1. Form an equation to solve for $y$.
2. Hence, find the required number of boys.

From Class Activity 1, we can summarize the steps involved in problem solving with linear equations as follows:

```
ster (1) Read the question carefully and identify the unknown quantity.
ster (2) Use a letter to represent the unknown quantity (e.g. x).
step (3) Express other quantities in terms of }x\mathrm{ .
ster (4) Form an equation based on the given information.
ster (3) Solve the equation.
step (6) Write down the answer statement.
```

Note: It is a good practice to check whether the solution you have obtained satisfies the conditions in the original problem. For instance, some problems may require the solution to be a positive integer. If we get a solution $x=-\frac{2}{3}$, it should be rejected.

The sum of three consecutive integers is 111 . Find the integers.

Solution
step (1) We are going to find the three integers.
step (2) Let $x$ be the smallest integer.
step (3) Middle integer $=x+1$
Largest integer $=x+2$

step (4) Sum of three integers $=111$

$$
\therefore x+(x+1)+(x+2)=111 \quad \therefore \text { the equation is }
$$

$$
3 x+3=111 \quad 3 x+3=111
$$

step (5)

$$
3 x=111-3
$$

$$
3 x=108
$$

$$
x=\frac{108}{3}
$$

$$
\therefore x=36
$$

step (c) The three integers are 36,37 , and 38.

Try It! 13 The sum of three consecutive integers is 144 . Find the integers.

## REMARKS

It is important to set just one variable $x$ and express the other two integers in terms of $x$. Otherwise, there would be too many variables to be solved.

| Check: |  |
| :---: | :---: |
| When | $x=36$, |
|  | $x+1=37$ |
| and | $x+2=38$. |
|  | + $38=111$ |
| The solu | = 36 is |

Example 14 Mrs. Lee is 3 times as old as her daughter. In 5 years' time, the sum of their ages will be 62 years. Find the daughter's present age.

Solution Let the daughter's present age be $x$ years. Mrs. Lee's present age $=3 x$ years.
In 5 years' time,
Mrs. Lee's age $=(3 x+5)$ years.
In 5 years' time, her daughter's age $=(x+5)$ years.
Since their total age in 5 years' time will be 62 , we have

$$
\begin{aligned}
(3 x+5)+(x+5) & =62 \\
3 x+5+x+5 & =62 \\
4 x & =62-10 \\
x & =\frac{52}{4} \\
\therefore x & =13
\end{aligned}
$$

Model Method
Present age:


Age five years later:

$\therefore$ the equation is $4 x+10=62$.

Try It! 14 Mr. Taylor is 4 times as old as his son. Four years ago, the sum of their ages was 37 years. Find the son's present age.

Example 15 The price of a table is $\$ 100$ less than 6 times the price of a chair. A similar set of one table and 4 chairs is priced at $\$ 1,400$. Find the price of a chair.

Solution Let the price of a chair be $\$ x$.
Price of a table $=\$(6 x-100)$
Price of a table and 4 chairs $=\$ 1,400$

## Model Method

$(6 x-100)+4 x=1,400$
$6 x-100+4 x=1,400$ $10 x=1,400+100$
$x=\frac{1,500}{10}$


$$
\therefore x=150
$$

Hence, the price of a chair is $\$ 150$.

Try It! 15 The price of a book is $\$ 1$ more than twice the price of a pen. The total price of 5 books and 4 pens is $\$ 47$. Find the price of a pen and of a book.

## REVIEW EXERCISE 5

1. Solve the following equations.
(a) $13 x-22=30$
(b) $2(5 x-8)+6=11$
(c) $\frac{2 x}{3}+\frac{x}{5}=13$
(d) $1-\frac{4}{7} x=23+x$
(e) $\frac{4 x-5}{2}=\frac{7 x-3}{9}$
(f) $\frac{x-4}{3}-\frac{2 x+1}{6}=\frac{5 x-1}{2}$
(g) $\frac{2}{x-7}=6$
(h) $\frac{4 x-1}{5 x+1}=\frac{5}{7}$
2. Given the formula $D=b^{2}-4 a c$,
(a) find the value of $D$ when $a=1, b=-5$, and $c=3$,
(b) find the value of $c$ when $a=2, b=3$, and $D=49$.
3. Given the formula $S=\frac{n(a+b)}{2}$,
(a) find the value of $S$ when $a=1, b=25$, and $n=12$,
(b) find the value of $a$ when $b=41, n=15$, and $S=330$.
4. The lengths of the sides of a triangle are $(2 x+1) \mathrm{cm},(3 x+2) \mathrm{cm}$, and $(4 x-1) \mathrm{cm}$.
(a) Find the perimeter of the triangle in terms of $x$.
(b) If the perimeter of the triangle is 47 cm , find the value of $x$.

5. Peter has 96 stamps and Sam has 63. How many stamps should Sam give Peter so that Peter will have twice as many stamps as Sam?
6. A boy is 26 years younger than his father. In 3 years' time, his age will be $\frac{1}{3}$ his father's age. Find the boy's present age.
7. The price of a skirt is $\$ 25$ more than the price of a T-shirt. The total price of 3 skirts and 8 T -shirts is $\$ 339$. Find the price of a skirt.

8. In a certain week, the amount of time Lisa spent on watching television was 3 hours more than twice the time she spent on doing her mathematics homework. If the total time she spent on these two activities was 30 hours in that week, how many hours did Lisa spend on doing her mathematics homework?
9. The number of books in a class library is 17 more than 3 times the number of students in the class. If 5 students are absent, each student can borrow exactly 4 books from the library. Find the number of students in the class.
10. A number is 4 times greater than another number. By subtracting 3 from each number, the first number becomes 5 times greater than the second. What are the two numbers?

## EXTENロ YロUR LEARNING CURVE

## Matchstick Triangle Patterns

Johnny uses matchsticks to form a pattern of triangles as shown below.


Suppose $m$ matchsticks are required to form $n$ triangles.
(a) Copy and complete the following table.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $m$ | 3 |  |  |  |  |  |

(b) Find a formula connecting $m$ and $n$.
(c) How many matchsticks are required to form 100 triangles?
(d) How many triangles can be formed with 2,005 matchsticks?
(e) Suppose the area of a triangle is $\sqrt{3} \mathrm{~cm}^{2}$. Find the total area of the triangles formed in (d). Give your answer correct to the closest whole numbers.

## WRJITE IN YOUR JDURNAL

Did you find the general strategy used to solve word problems on linear equations helpful? Why or why not?

## RATIU, RATEANB SFEED



## LET'S LEARN TTO..

(1) understand the concept of ratio
(2) state the relationship between ratio and fraction
(3) solve problems involving ratio
(4) understand the concepts of rate and average rate
(5) solve problems involving rate

6 understand the concepts of uniform speed and average speed

5 solve problems involving speed


The density of a material is defined as its mass per unit volume, that is, density $=\frac{\text { mass }}{\text { volume }}$. Because the ratio of the density of pure ice to that of sea water is 9 : 10, typically, only a small part of a giant iceberg is above water. The shape of the underwater portion can be difficult to judge by looking at the portion above the surface. Do you know what fraction of the volume of the iceberg is exposed?

### 6.1 Ratios Involving Rational Numbers

## A Meaning of Ratio

We have learned the idea of a ratio in the previous grade. Let us recall its meaning.

Given any two similar quantities, $a$ and $b$, the ratio of $a$ to $b$ (denoted by $a: b$ ) is defined as

$$
a: b=\frac{a}{b} \text {, where } b \neq 0
$$

A ratio is a comparison of two similar quantities. In this section, we will limit our discussion on ratios to those involving rational numbers.

## Example 1

Solution
There are 16 boys and 20 girls in a class. Find the ratio of
(a) the number of boys to the number of girls,
(b) the number of girls to the total number of students in the class.
(a) Ratio of the number of boys to the number of girls

$$
=16: 20
$$

$$
=4: 5
$$

$$
16: 20=\frac{16}{20}=\frac{4 \times 4}{4 \times 5}=\frac{4}{5}
$$

(b) Total number of students $=16+20$

$$
=36
$$

Ratio of the number of girls to the total number of students
= $20: 36$
$=5: 9$

Note: Since a ratio can be expressed as a fraction, we will reduce the ratio to its simplest form $a: b$, where $a$ and $b$ have no common factors except for 1 .

Try lt! (1) A bag consists of 25 green balls and 15 red balls. Find the ratio of
(a) the number of green balls to the number of red balls,
(b) the number of red balls to the total number of balls in the bag.

If a cheetah can run at a uniform speed of $30 \mathrm{~m} / \mathrm{s}$ for 40 seconds, then its distance traveled in $40 \mathrm{~s}=30 \times 40$

$$
=1,200 \mathrm{~m} .
$$

However, in real-life situations, it is difficult to maintain a uniform speed throughout a period of time. It is more realistic to use average speed in our calculations.

$$
\text { Average speed }=\frac{\text { Total distance covered }}{\text { Total time taken }}
$$

## MATH BITS

A railway line and a road run parallel. Every morning, Mr. Perry cycles to work along this road and meets a train traveling in the same direction at a junction. One day, he was late by 25 minutes and the train had traveled 6 kilometers beyond the junction. What was the speed of the train if Mr. Perry cycled at an average speed of $12 \mathrm{~km} / \mathrm{h}$ on that day?

A cheetah can only maintain its high speed of $30 \mathrm{~m} / \mathrm{s}$ for a distance of about 400 meters to 800 meters. If it runs 1,200 meters in 50 seconds,
its average speed $=\frac{1,200 \mathrm{~m}}{50 \mathrm{~s}}$

$$
=24 \mathrm{~m} / \mathrm{s} .
$$

Note that the cheetah's speed at any moment may be higher or lower than $24 \mathrm{~m} / \mathrm{s}$.

## CLASS ACIVIJン?

The speed of a car for the first 2 hours of a journey is 50 miles per hour. Its speed for the next hour is 74 miles per hour. What is its average speed for the whole journey?

## Questions

1. What would be a more accurate term to describe the word 'speed' in the first sentence? Why?
2. Which one of the two solutions below is the correct method of obtaining the average speed for the whole journey, and why?

## Solution (i):

Average speed $=\frac{1}{2}(50+74)$

$$
=62 \mathrm{mi} / \mathrm{hr}
$$

## Solution (ii):

Total distance traveled $=50 \times 2+74$

$$
=174 \text { miles }
$$

Total time taken $=3 \mathrm{hr}$
Average speed $=174 \mathrm{mi} \div 3 \mathrm{hr}$
$=58 \mathrm{mi} / \mathrm{hr}$

### 7.2 Reverse Percentages

Let us consider the following examples involving reverse percentages.

## Example 7

In a box, $15 \%$ of the balls are green. If there are 54 green balls, find the number of balls in the box.

Solution
Let $n$ be the number of balls in the box. Then $15 \%$ of $n$ is 54 .

Alternative Method:
$\therefore 15 \% \times n=54 \quad 15 \% \rightarrow 54$ $n=54 \div 15 \%$
$1 \% \rightarrow \frac{54}{15}$
$=54 \div \frac{15}{100}$
$=54 \times \frac{100}{15}$
$100 \% \rightarrow \frac{54}{15} \times 100$
$=360$
The number of balls in the box is 360 .

Note: When a calculator is used to evaluate an expression involving a percentage, it is more convenient to convert the percentage to a decimal.

Try It! 7 In a class, $25 \%$ of the students wear glasses. If there are 8 students wearing glasses, find the number of students in the class.

Example 8 In a library, the fine for not returning a book on loan is $125 \%$ of the price of the book. If the fine for a book that was not returned was $\$ 90$, find the price of the book.

Solution Let $\$ p$ be the price of the book.
Then $125 \%$ of $p$ is $\$ 90$.
$\therefore 125 \% \times p=90$

$$
p=90 \div 125 \%
$$

$$
=90 \div \frac{125}{100}
$$

$$
=72
$$

The price of the book is $\$ 72$.

Try lt! 8 Kumar was fined $\$ 187$ for traffic speeding. If the fine was $110 \%$ of his daily wage, find Kumar's daily wage.

## EXERCISE 7.2

## BASICPRACTICE

1. Find the unknown quantity in each case.
(a) $30 \%$ of $a$ is 18 .
(b) $37.5 \%$ of $\$ b$ is $\$ 108$.
(c) $22 \frac{2}{9} \%$ of $c \mathrm{~kg}$ is 44 kg .
(d) $150 \%$ of $d \mathrm{~cm}^{2}$ is $126 \mathrm{~cm}^{2}$.
(e) $0.5 \%$ of $e{ }^{\circ} \mathrm{C}$ is $7^{\circ} \mathrm{C}$.
(f) $\frac{1}{3} \%$ of $f$ hours is 12 hours.

## FURTHERPRACTICE

2. Adam attempts $65 \%$ of the questions in a test. If he attempts 52 questions, find the total number of questions in the test.
3. $45 \%$ of the members in a council are women. There are 72 female council members. Find
(a) the total number of council members,
(b) the number of male council members.
4. $85 \%$ of the customers of a supermarket were residents of the neighborhood. Given that 2,380 of the customers on a particular day were residents, find
(a) the total number of customers,
(b) the number of customers who were not residents of the neighborhood on that day.
5. After cycling 18 km at an average speed of $12 \mathrm{~km} / \mathrm{hr}$, Lucy finds that she still has to cycle $55 \%$ of the total distance. She then completes the rest of her journey at an average speed of $16.5 \mathrm{~km} / \mathrm{hr}$. Find
(a) the total distance of her journey,
(b) the remaining distance she needs to cycle to complete the journey,
(c) the time taken for the whole journey,
(d) the average speed for the whole journey.

## MATH@WロRIK

6. A drink stall in the mall sells bottled water, juice and soda. On a particular day, it sold 175 out of its 200 bottled water, $85 \%$ of its 220 bottled juice and 180 bottles of soda which is $80 \%$ of its original stock of bottled soda.
(a) Which drink has the highest number of bottles sold? How many bottles of this drink were sold?
(b) Which drink has sold the highest percentage of its stock? What was the percent figure?
(c) Which drink has the greatest number of bottles in its original stock? What was the figure?
7. $38 \%$ of Sumiko's music CDs are Chinese music CDs, $44 \%$ of them are English music CDs and the rest are Japanese music CDs. There are 45 Japanese music CDs.
(a) Find the total number of CDs in the collection.
(b) Find the number of Chinese music CDs.
(c) If $60 \%$ of the English music CDs are CDs with pop songs, find the number of CDs with English pop songs.

## BRAIN WIRIKS

8. (a) Is $60 \%$ of 30 equal to $30 \%$ of 60 ? Explain your answer. Generalize your findings using variables $x$ and $y$.
(b) If $60 \%$ of $x$ is 30 and $30 \%$ of $y$ is 60 , is $x$ equal to $y$ ? Explain your answer.
(c) Write a real-life problem that involves the mathematics in (a).

Meaning of Percentage

$$
\begin{aligned}
n \% & =\frac{n}{100} \\
1 \% & =\frac{1}{100} \\
100 \% & =1
\end{aligned}
$$

## Expressing One Quantity as a Percentage of Another and Reverse Percentage

If $a$ is $n \%$ of $b$, then $\quad a=\frac{n}{100} \times b$
and $\quad b=\frac{100 a}{n}$.

## Percentage Increase

Increase $=$ Increased value - Original value Percentage increase $=\frac{\text { Increase }}{\text { Original value }} \times 100 \%$

Increased value $=(100 \%+$ Increase $\%) \times$ Original value

## Percentage Decrease

Decrease $=$ Original value - Decreased value
Percentage decrease $=\frac{\text { Decrease }}{\text { Original value }} \times 100 \%$
Decreased value $=(100 \%-$ Decrease $\%) \times$ Original value

## Discount

Discount $=$ Marked price - Selling price Percentage discount $=\frac{\text { Discount }}{\text { Marked price }} \times 100 \%$

Selling price $=(100 \%-$ Discount \% $)$
$\times$ Marked price

Tax
Tax $=$ Tax rate $\times$ Cost

Objective: To explore the properties of angles at a point, complementary angles, supplementary angles and vertically opposite angles.

(You may wish to refer to the navigational guide for Sketchpad at the back of this book before doing this activity.)

## Tasks

(a) Start Sketchpad.
(b) Mark a point $P$ anywhere on the screen and draw an acute angle $x$, an obtuse angle $y$ and a reflex angle $z$ with the common vertex $P$ such that each angle is adjacent to the other two angles.
(c) Measure $\angle x, \angle y$, and $\angle z$ and find their sum.
(d) Construct a right angle, $\angle L M N$, and a pair of complementary angles.
(e) Construct a pair of supplementary angles.
(f) Draw two intersecting straight lines $A B$ and $C D$ as shown above.
(g) Measure all the four angles $\angle a, \angle b, \angle c$, and $\angle d$ at their point of intersection $E$.
(h) Drag one of the lines and observe the changes in these angles and how they are related.

## Questions

1. What can you say about the sum of $m \angle x, m \angle y$, and $m \angle z$ ?
2. Describe your way of drawing a pair of complementary angles.
3. Describe your way of drawing a pair of supplementary angles.
4. What can you say about the relationship between $\angle a$ and $\angle b$ ?
5. What can you say about the relationship between $\angle a$ and $\angle c$ ?
