

REMARKS To multiply or divide fractions, the denominators need not be converted to the same number.

3.3 Writing Algebraic Expressions to Represent Real-world Situations

We may use algebraic expressions and formulas to express the relationship between two or more quantities in our daily life. Let us see some examples.



From the above class activity, we observe that 3(x + y) = 3x + 3y. In general, we have the **distributive law** of multiplication over addition:

a(x+y) = ax + ay

We say that a(x + y) is expanded to ax + ay. The expressions a(x + y) and ax + ay are equivalent expressions.

This law can be generalized and applied as follows:

 (x + y)a = a(x + y) = ax + ay = xa + ya Multiplication can be distributed over addition from the right.
 a(x - y) = a[x + (-y)] = ax + a(-y) = ax - ay Multiplication can be distributed over subtraction.

3.
$$a(x + y + z) = ax + ay + az$$

Multiplication can be distributed over several terms.

Example 🕃 Expand each expression by removing the parentheses. (a) a(3b + c)**(b)** -x(2y-z)Solution (a) a(3b + c) = 3ab + ac**(b)** -x(2y-z) = -x[2y + (-z)]= (-x)(2y) + (-x)(-z)RECALL = -2xy + xz(a)(b) = ab(-a)(-b) = abTry It! 3 Expand each expression by removing the parentheses. (a)(-b) = -ab(a) a(2b - 3c)**(b)** -x(-5y + z)(-a)(b) = -ab

Note: The distributive law is applicable when removing parentheses in algebraic expressions such as x - (a - b). This expression can be interpreted as x + (-1)(a - b).

$$\begin{aligned} x - (a - b) &= x + (-1)(a - b) \\ &= x + (-1)[a + (-b)] \\ &= x + (-1)(a) + (-1)(-b) \\ &= x - a + b \end{aligned}$$



Objective: To formulate linear equations to solve problems.

Questions

A group of boys and girls planted a total of 148 trees. Each boy planted 7 trees, and each girl planted 5 trees. There were 4 more boys than girls in the group. How many boys were there in the group?

- 1. Identify the unknown quantity that you are required to find in the problem.
- 2. Use a letter (e.g. *x*) to represent the unknown quantity.
- **3.** Express other quantities in terms of the letter that represents the unknown quantity. In this case, express each of the following in terms of x.
 - (a) the number of girls
 - (b) the number of trees planted by all the boys
 - (c) the number of trees planted by all the girls
- 4. Form an equation required to solve the problem.
- **5.** Solve the equation.
- 6. Write down the answer statement.

Alternative Method:

We may use a model to formulate the problem.

Let the number of girls be *y*.

Girls:	у	
Boys:	у	4

Consider the numbers of trees planted by the girls and the boys, denoted by GT and BT respectively, using the following model.



- **1.** Form an equation to solve for *y*.
- 2. Hence, find the required number of boys.

From Class Activity 1, we can summarize the steps involved in problem solving with linear equations as follows:

- **STEP ()** Read the question carefully and identify the unknown quantity.
- **STEP 2** Use a letter to represent the unknown quantity (e.g. *x*).
- **STEP 3** Express other quantities in terms of x.
- **STEP ()** Form an equation based on the given information.
- **Solve the equation.**
- **STEP 6** Write down the answer statement.
- **Note:** It is a good practice to check whether the solution you have obtained satisfies the conditions in the original problem. For instance, some problems may require the solution to be a positive integer. If we get a solution $x = -\frac{2}{3}$, it should be rejected.





REVIEW EXERCISE 5

- 1. Solve the following equations.
 - (a) 13x 22 = 30
 - **(b)** 2(5x-8)+6=11
 - (c) $\frac{2x}{3} + \frac{x}{5} = 13$

(d)
$$1 - \frac{4}{7}x = 23 + x$$

(e)
$$\frac{4x-5}{2} = \frac{7x-3}{9}$$

(f) $\frac{x-4}{3} - \frac{2x+1}{6} = \frac{5x-3}{2}$

(g)
$$\frac{2}{x-7} = 6$$

(1) $\frac{4x-1}{5}$

- (h) $\frac{1}{5x+1} = \frac{3}{7}$
- **2.** Given the formula $D = b^2 4ac$,
 - (a) find the value of *D* when a = 1, b = -5, and c = 3,
 - (b) find the value of c when a = 2, b = 3, and D = 49.
- **3.** Given the formula $S = \frac{n(a+b)}{2}$,
 - (a) find the value of S when a = 1, b = 25, and n = 12,
 - (b) find the value of a when b = 41, n = 15, and S = 330.
- 4. The lengths of the sides of a triangle are (2x + 1) cm, (3x + 2) cm, and (4x 1) cm.
 - (a) Find the perimeter of the triangle in terms of x.
 - (b) If the perimeter of the triangle is 47 cm, find the value of x.



5. Peter has 96 stamps and Sam has 63. How many stamps should Sam give Peter so that Peter will have twice as many stamps as Sam?

- 6. A boy is 26 years younger than his father. In 3 years' time, his age will be $\frac{1}{3}$ his father's age. Find the boy's present age.
- 7. The price of a skirt is \$25 more than the price of a T-shirt. The total price of 3 skirts and 8 T-shirts is \$339. Find the price of a skirt.



- 8. In a certain week, the amount of time Lisa spent on watching television was 3 hours more than twice the time she spent on doing her mathematics homework. If the total time she spent on these two activities was 30 hours in that week, how many hours did Lisa spend on doing her mathematics homework?
- **9.** The number of books in a class library is 17 more than 3 times the number of students in the class. If 5 students are absent, each student can borrow exactly 4 books from the library. Find the number of students in the class.
- **10.** A number is 4 times greater than another number. By subtracting 3 from each number, the first number becomes 5 times greater than the second. What are the two numbers?



EXTEND YOUR LEARNING CURVE

Matchstick Triangle Patterns

Johnny uses matchsticks to form a pattern of triangles as shown below.



Suppose m matchesticks are required to form n triangles.

(a) Copy and complete the following table.

n	1	2	3	4	5	6
m	3					

- (b) Find a formula connecting *m* and *n*.
- (c) How many matchsticks are required to form 100 triangles?
- (d) How many triangles can be formed with 2,005 matchsticks?
- (e) Suppose the area of a triangle is $\sqrt{3}$ cm². Find the total area of the triangles formed in (d). Give your answer correct to the closest whole numbers.

WRITE IN YOUR JOURNAL

Did you find the general strategy used to solve word problems on linear equations helpful? Why or why not?





- **3** solve problems involving ratio
- 4 understand the concepts of rate and average rate
- 5 solve problems involving rate
- 6 understand the concepts of uniform speed and average speed
- 5 solve problems involving speed

ratio of the density of pure ice to that of sea water is 9 : 10, typically, only a small part of a giant iceberg is above water. The shape of the underwater portion can be difficult to judge by looking at the portion above the surface. Do you know what fraction of the volume of the iceberg is exposed?

6.1 Ratios Involving Rational Numbers

A Meaning of Ratio

We have learned the idea of a ratio in the previous grade. Let us recall its meaning.

Given any two similar quantities, a and b, the ratio of a to b (denoted by a : b) is defined as

 $a: b = \frac{a}{b}$, where $b \neq 0$.

A ratio is a comparison of two similar quantities. In this section, we will limit our discussion on ratios to those involving rational numbers.

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If a cheetah can run at a uniform speed of 30 m/s for 40 seconds, then its distance traveled in 40 s = 30×40 = 1 200 m

= 1,200 m.

However, in real-life situations, it is difficult to maintain a uniform speed throughout a period of time. It is more realistic to use **average speed** in our calculations.



A railway line and a road run parallel. Every morning, Mr. Perry cycles to work along this road and meets a train traveling in the same direction at a junction. One day, he was late by 25 minutes and the train had traveled 6 kilometers beyond the junction. What was the speed of the train if Mr. Perry cycled at an average speed of 12 km/h on that day?

MATH BITS

A cheetah can only maintain its high speed of 30 m/s for a distance of about 400 meters to 800 meters. If it runs 1,200 meters in 50 seconds,

its average speed = $\frac{1,200 \text{ m}}{50 \text{ s}}$ = 24 m/s.

Note that the cheetah's speed at any moment may be higher or lower than 24 m/s.



The speed of a car for the first 2 hours of a journey is 50 miles per hour. Its speed for the next hour is 74 miles per hour. What is its average speed for the whole journey?

Questions

- **1.** What would be a more accurate term to describe the word 'speed' in the first sentence? Why?
- **2.** Which one of the two solutions below is the correct method of obtaining the average speed for the whole journey, and why?

Solution (i):
Average speed
$$=\frac{1}{2}(50 + 74)$$

 $= 62 \text{ mi/hr}$
Solution (ii):
Total distance traveled $= 50 \times 2 + 74$
 $= 174 \text{ miles}$
Total time taken $= 3 \text{ hr}$
Average speed $= 174 \text{ mi} \div 3 \text{ hr}$
 $= 58 \text{ mi/hr}$

7.2 Reverse Percentages

Let us consider the following examples involving reverse percentages.

Example 7	In a box, 15% of the balls are green. If there are 54 green	
	balls, find the number of balls in the box.	
• Solution	Let <i>n</i> be the number of balls in the box. Then 15% of <i>n</i> is 54. Alternative Method: $\therefore 15\% \times n = 54 \qquad 15\% \rightarrow 54$ $n = 54 \div 15\%$ $= 54 \div \frac{15}{100} \qquad 1\% \rightarrow \frac{54}{15}$ $= 54 \times \frac{100}{15} \qquad = 360$ $= 360$ The number of balls in the box is 360.	
	Note: When a calculator is used to evaluate an expression involving a percentage, it is more convenient to convert the percentage to a decimal.	
Try It! 7	In a class, 25% of the students wear glasses. If there are 8 students wearing glasses, find the number of students in the class.	
Example 8	In a library, the fine for not returning a book on loan is 125% of the price of the book. If the fine for a book that was not returned was \$90, find the price of the book.	
• Solution	Let \$p be the price of the book. Then 125% of p is \$90. $\therefore 125\% \times p = 90$ $p = 90 \div 125\%$ $= 90 \div \frac{125}{100}$ = 72 The price of the book is \$72.	
Try it! 8	Kumar was fined \$187 for traffic speeding. If the fine was 110% of his daily wage, find Kumar's daily wage.	

REMARKS Observe that $15\% \times n = 54$ is a linear equation with unknown *n*.

EXERCISE 7.2



- Find the unknown quantity in each case.
 (a) 30% of a is 18.
 - (**b**) 37.5% of \$*b* is \$108.
 - (c) $22\frac{2}{9}\%$ of c kg is 44 kg.
 - (d) 150% of $d \text{ cm}^2$ is 126 cm^2 .
 - (e) 0.5% of $e \, ^\circ C$ is 7 $^\circ C$.
 - (f) $\frac{1}{2}$ % of f hours is 12 hours.



- 2. Adam attempts 65% of the questions in a test. If he attempts 52 questions, find the total number of questions in the test.
- **3.** 45% of the members in a council are women. There are 72 female council members. Find
 - (a) the total number of council members,
 - (b) the number of male council members.
- **4.** 85% of the customers of a supermarket were residents of the neighborhood. Given that 2,380 of the customers on a particular day were residents, find
 - (a) the total number of customers,
 - (b) the number of customers who were not residents of the neighborhood on that day.
- 5. After cycling 18 km at an average speed of 12 km/hr, Lucy finds that she still has to cycle 55% of the total distance. She then completes the rest of her journey at an average speed of 16.5 km/hr. Find
 - (a) the total distance of her journey,
 - (b) the remaining distance she needs to cycle to complete the journey,
 - (c) the time taken for the whole journey,
 - (d) the average speed for the whole journey.



- 6. A drink stall in the mall sells bottled water, juice and soda. On a particular day, it sold 175 out of its 200 bottled water, 85% of its 220 bottled juice and 180 bottles of soda which is 80% of its original stock of bottled soda.
 - (a) Which drink has the highest number of bottles sold? How many bottles of this drink were sold?
 - (b) Which drink has sold the highest percentage of its stock? What was the percent figure?
 - (c) Which drink has the greatest number of bottles in its original stock? What was the figure?
- 38% of Sumiko's music CDs are Chinese music CDs, 44% of them are English music CDs and the rest are Japanese music CDs. There are 45 Japanese music CDs.
 - (a) Find the total number of CDs in the collection.
 - (b) Find the number of Chinese music CDs.
 - (c) If 60% of the English music CDs are CDs with pop songs, find the number of CDs with English pop songs.

BRAINWORKS

- 8. (a) Is 60% of 30 equal to 30% of 60? Explain your answer. Generalize your findings using variables x and y.
 - (b) If 60% of x is 30 and 30% of y is 60, is x equal to y? Explain your answer.
 - (c) Write a real-life problem that involves the mathematics in (a).

IN A MUT Meaning of Percentage

$$n\% = \frac{n}{100}$$

 $1\% = \frac{1}{100}$
 $100\% = 1$

Expressing One Quantity as a Percentage of Another and Reverse Percentage

If a is n% of b, then $a = \frac{n}{100} \times b$ and $b = \frac{100a}{n}$.

Percentage Increase

Increase = Increased value - Original value

Percentage increase= $\frac{\text{Increase}}{\text{Original value}} \times 100\%$

Increased value = $(100\% + \text{Increase }\%) \times \text{Original value}$

Percentage Decrease

Decrease = Original value – Decreased value

Percentage decrease = $\frac{\text{Decrease}}{\text{Original value}} \times 100\%$

Decreased value = $(100\% - Decrease \%) \times Original value$

Discount

Discount = Marked price – Selling price

 $Percentage discount = \frac{Discount}{Marked price} \times 100\%$

Selling price = (100% - Discount %)× Marked price Tax

 $Tax = Tax rate \times Cost$

CLASS ACTIVITY 2

Objective: To explore the properties of angles at a point, complementary angles, supplementary angles and vertically opposite angles.



(You may wish to refer to the navigational guide for Sketchpad at the back of this book before doing this activity.)

Tasks

- (a) Start Sketchpad.
- (b) Mark a point *P* anywhere on the screen and draw an acute angle *x*, an obtuse angle *y* and a reflex angle *z* with the common vertex *P* such that each angle is adjacent to the other two angles.
- (c) Measure $\angle x$, $\angle y$, and $\angle z$ and find their sum.
- (d) Construct a right angle, $\angle LMN$, and a pair of complementary angles.
- (e) Construct a pair of supplementary angles.
- (f) Draw two intersecting straight lines AB and CD as shown above.
- (g) Measure all the four angles $\angle a$, $\angle b$, $\angle c$, and $\angle d$ at their point of intersection *E*.
- (h) Drag one of the lines and observe the changes in these angles and how they are related.

Questions

- **1.** What can you say about the sum of $m \angle x$, $m \angle y$, and $m \angle z$?
- 2. Describe your way of drawing a pair of complementary angles.
- 3. Describe your way of drawing a pair of supplementary angles.
- **4.** What can you say about the relationship between $\angle a$ and $\angle b$?
- **5.** What can you say about the relationship between $\angle a$ and $\angle c$?